Convergence of Improper Integrals.

Definition - Proper Integral

The definite integral $\int_a^a f(x)\,dx$ is called proper integral if the range of integration is finite and the integrand $f(x)$ is bounded.

Definition - Improper integral

An improper integral is a definite integral that has either or both limits infinite or an integrand that approaches ∞ at one or more points in the range of integrals.

There are two types of improper integrals.

(i) $\int_a^b f(x)\,dx$

The limit a or b both are infinite.

(ii) the function $f(x)$ has one or more points of discontinuity in the internal $[a,b]$.

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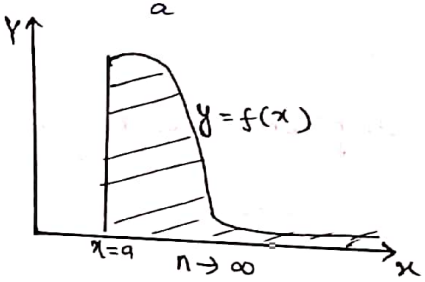
Type 1 Let $f(x)$ be a continuous function on the interval $[a, \infty)$, we define the improper integral as -

In order to integrate over the infinite domain $[a, \infty)$ we consider the limit of the form

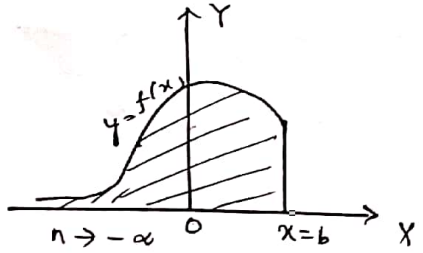
$$\int_a^{\infty} f(x)\,dx = \lim_{n\to\infty} \int_a^{n} f(x)\,dx$$



Similarly, if a continuous function $f(x)$ is given on the interval $(-\infty, b]$

The improper integral $f(x)$ is defined as

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{n\to-\infty} \int_n^{b} f(x)\,dx$$



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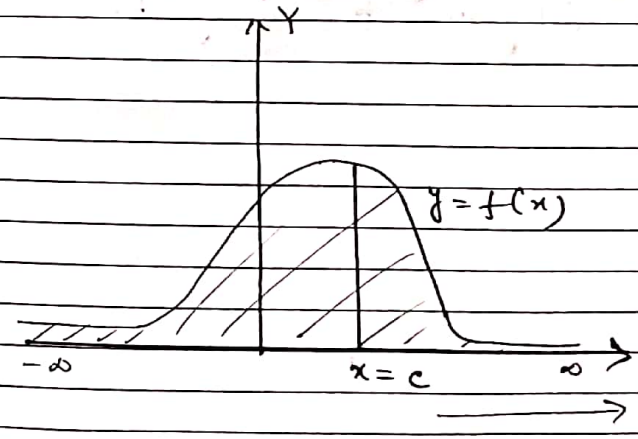
If these limit exist and are finite then we say that the improper integrals are cgt. otherwise the integrals are dgt.

An improper integral might have two infinite limits.

In this case, we can pick an arbitrary point c & break the integral up there. We obtain two improper integral each with one infinite limit.

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_{c}^{\infty} f(x)\,dx$$



If for some real number c, both of the integral in the R.H.S. are convergent, then we say that the integral $\int_{-\infty}^{\infty} f(x)\,dx$ is also convergent; else divergent.

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Convergence and Divergence of Improper Integral

Let us consider $f(x)$ which exhibits a Type(i) or Type(ii) behaviour on the interval $[a, b]$.

Therefore we have two cases :—

(a) The limit exists (and is a number). In this case we say that the improper integral is convergent.

(b) If limit does not exist or it is infinite, then we say that improper integral is divergent.

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